Reg No : .... Name : ....

MAR ATHANASIUS COLLEGE OF ENGINEERING, KOTHAMANGALAM

(Govt. Aided Autonomous Institution)

Third Semester B.Tech Degree (SAY) Examination, February, 2026

Course Code: B24MA2T03A

Course Name: COMPLEX VARIABLES AND APPLICATIONS OF PDE

Max. Marks: 100 Duration:3 Hours

PART - A: 30 Marks (10 Qns x 3 Marks)

Answer all questions

1. Form the pde from the equation $z = f(x^2 - y^2)$ where f is an arbitrary function.

2. Solve $p - q = log(x + y)$.

3. Using D'Alemberts' method, find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = k(\sin x - \sin 3x)$.

4. Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ using the method of seperation of variables when the seperation constant $k > 0$.

5. Find the image of $x = c$ under the mapping $w = e^z$.

6. Check the differentiability of the function $f(z) = e^{-x} \cos y - ie^{-x} \sin y$.

7. Evaluate $\int_C \frac{z^2+2z+3}{z^2-1} dz$ where C is $|z - 1| = 1$.

8. Evaluate $\int_{-\pi i}^{\pi i} \cos z \, dz$.

9. Find the Maclaurin series of $f(z) = \sin z$.

10. Find all singular points and residue of the function $cosec\, z$.

PART - B: 70 Marks (5 Qns x 14 Marks)

Answer any One full question from each module.

Module 1

11. a. Solve $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method. (7 Marks)

    b. Solve $y^2p - xyq = xz$. (7 Marks)

OR

12. a. Form the pde by eliminating the arbitrary functions from $z = f_1(x)f_2(y)$. (7 Marks)

    b. Solve $p - 2q = 3x^2 sin(y + 2x)$. (7 Marks)

Module 2

13. a. A homogeneous rod of heat conducting material of length 100 cm has its ends kept at 0^0 temperature and the initial temperature is (7 Marks)

$$u(x,0) = \begin{cases} x & if \quad 0 \le x \le 50 \\ 100 - x & if \quad 50 \le x \le 100 \end{cases}.$$

Find the temperature $u(x,t)$ at any time t.

b. A tightly stretched string of length $1cm$ is fastened at both ends. Find the displacement of the string if it is released from rest from the position $\sin(\pi x) + 5\sin(3\pi x)$. (7 Marks)

OR

14. a. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = asin(\frac{\pi x}{l})$ from which it is released at time $t = 0$. Find the displacement $y(x,t)$. (7 Marks)

b. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0,\ u(\pi,t) = 0,\ t \ge 0,\ u(x,0) = \pi x - x^2, 0 \le x \le \pi$. (7 Marks)

Module 3

15. a. Find the analytic function whose imaginary part is $v = \cos x \sinh y$. (7 Marks)

b. Find the image of the circle $|z-1| = 1$ under the mapping $w = \frac{1}{z}$. (7 Marks)

OR

16. a. Show that $u = e^x \cos y$ is a harmonic function and hence find the analytic function of which it is the real part. (7 Marks)

b. Prove that the function $\sinh z$ is analytic and hence find the derivative. (7 Marks)

Module 4

17. a. Evaluate $\oint_C \frac{z^2+z+1}{(z-1)^2}\, dz$ where C is the circle $|z| = 3$ using Cauchy's integral formula. (7 Marks)

b. Integrate $\oint_C Im(z^2)dz$ where C is the triangle with vertices $0, 1$ and i taken in counter-clockwise direction. (7 Marks)

OR

18. a. Evaluate $\int_C \frac{3z^2+z}{(z^2-9)(z-1)}\, dz$ where C is the circle $|z-1| = 1$. (7 Marks)

b. Evaluate $\int_0^{1+i} (x^2 + ixy)dz$ along the curve $y = x^2$. (7 Marks)

Module 5

19. a. Find the Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ valid in (7 Marks)

i) $1 < |z| < 2$

ii) $|z| > 2$.

b. Find the residue of $f(z) = \frac{2z+1}{z^2-z-2}$ at its poles. (7 Marks)

OR

20. a. Expand $f(z) = \frac{z+1}{z-1}$ as a Taylor series about $z = -1$. (7 Marks)

b. Determine and classify the singular points of the following functions: (7 Marks)

i) $\frac{\sin z}{(z-\pi)^2}$

ii) $\tan z$

iii) $\frac{1}{(z-3)^2(z+5)}$.

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