QP Code:MA1T01-3 **(Pages: 3)**

Reg No : .... Name : ....

MAR ATHANASIUS COLLEGE OF ENGINEERING, KOTHAMANGALAM

(Govt. Aided Autonomous Institution)

First Semester B.Tech. Degree (SAY) Examination, February 2026

Course Code: B24MA1T01

Course Name: Linear algebra and multivariable calculus

Max. Marks: 100 Duration:3 Hours

PART - A: 30 Marks (10 Qns x 3 Marks)

Answer all questions

1. Write down the eigenvalues of $A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$. Hence find trace and determinant of A.

2. Find the matrix corresponding to the quadratic form $3x^2 - 2y^2 - z^2 - 4xy + 8xz + 12yz$.

3. Find the derivative of $w = x^2 + y^2$ with respect to t along the path $x = at^2, y = 2at$ using chain rule.

4. Find the differential dw of the function $w = \frac{xyz}{x+y+z}$.

5. Evaluate $\int_0^3 \int_0^{\sqrt{9-y^2}} 2y\, dx\, dy$.

6. Evaluate $\iint_R x dx dy$ where R is the region bounded by $y = x^2$ and the line $y = x$.

7. Find the speed of a particle moving along the given curve $x = 1 + 3t$, $y = 3 - 4t$, $z = 7 + 2t$ at $t = 2$.

8. Find the directional derivative of $\phi = x^2yz + 4x^2z$ in the direction of $2\hat{i} + \hat{j} - \hat{k}$ at $(0,1,1)$.

9. Apply Green's theorem for $\int_C \left(x^2y + y\right) dx + xy^2\, dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

10. Use divergence theorem to find the outward flux of the vector field $\vec{F} = 3x^3\,\hat{i} + 3y^3\,\hat{j}$ across the surface of the region that is enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$.

PART - B: 70 Marks (5 Qns x 14 Marks)

Answer any One full question from each module.

Module 1

11\. a. Find the eigenvalues and eigenvectors of the matrix (7 Marks)

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

b. Find the diagonal matrix of $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$. (7 Marks)

OR

12\. a. Solve the following system of equations using Gauss elimination method. (7 Marks)

$x + y + z = 6, x + 2y - 3z = -4, -x - 4y + 9z = 18.$

b. Show that the quadratic form $3x^2 + 22xy + 3y^2 = 0$ represents a pair of straight lines by transforming into principal axes form. (7 Marks)

Module 2

13\. a. If $w = x^2 + y^2 - z^2$ where $x = \rho sin\phi cos\theta$, $y = \rho sin\phi sin\theta$, $z = \rho cos\phi$, find $\frac{\partial w}{\partial \rho}$, $\frac{\partial w}{\partial \phi}$ and $\frac{\partial w}{\partial \theta}$ using chain rule. (7 Marks)

b. Locate all relative extrema and saddle points of (7 Marks)

$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$

OR

14\. a. A function $f(x, y) = x^2 + y^2$ is given with a local linear approximation $L(x, y) = 2x + 4y - 5$ to $f(x, y)$ at a point P. Determine the point P. (7 Marks)

b. If $u = f(x^2 + 2yz,\ y^2 + 2xz)$, prove that (7 Marks)

$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0.$

Module 3

15\. a. Using double integral to find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (7 Marks)

b. Find the mass of a triangular lamina with vertices $(0,0),(1,0)$ and $(0,2)$ and the density function is $\rho(x,y)=1+3x+y$. (7 Marks)

OR

16\. a. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. (7 Marks)

b. Use polar coordinates to evaluate $\int_{-1}^{1}\int_0^{\sqrt{1-x^2}} \left(x^2+y^2\right)^{\frac{3}{2}}\,dy\,dx$. (7 Marks)

Module 4

17\. a. Show that $\int_C \left(3x^2e^y\,dx + x^3e^y\,dy\right)$ is independent of the path and hence evaluate the integral from $(0,0)$ *to* $(3,2)$. (7 Marks)

b. Find the work done in moving a particle along a straight line from $(0,0,0)$ to $(2,1,3)$ under the force $\vec{F}=3x^2\,\hat{i}+(2xz-y)\,\hat{j}+z\,\hat{k}$. (7 Marks)

OR

18\. a. Show that the vector field $F(x,y)=2xy^3\,\hat{i}+3y^2x^2\,\hat{j}$ is conservative and find ϕ such that $\mathbf{F}=\nabla\phi$ and hence evaluate $\int_{(2,-2)}^{(-2,0)}\left(2xy^3\,dx+3y^2x^2\,dy\right)$. (7 Marks)

b. Find the divergence and curl of $F=x^3y^3z\hat{i}\;+\;xyz^3\hat{j}\;+\;xyz^2\hat{k}$ at $(1,1,1)$. (7 Marks)

Module 5

19\. a. Using Green's theorem find the work done by the force $\mathbf{F}(x,y)=\sqrt{y}\,\hat{i}+\sqrt{x}\,\hat{j}$ on a particle that moves counter clockwise once around the closed curve given by the equations $y=0, x=2$ and $y=\frac{x^3}{4}$. (7 Marks)

b. Evaluate $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where S is the surface of the cylinder $x^2+y^2=4$, $z=0, z=3$ where $F=(2x-y)\hat{i}+(2y-z)\hat{j}+z^2\hat{k}$. (7 Marks)

OR

20\. a. Using Stoke's theorem, evaluate $\int_C \vec{\mathbf{f}}.d\vec{\mathbf{r}}$ where $\vec{\mathbf{f}}=(z-y)\hat{\mathbf{i}}+(z+x)\hat{\mathbf{j}}-(x+y)\hat{\mathbf{k}}$ where C is the boundary of the paraboloid $z=9-x^2-y^2$ above the $xy-$ plane with upward orientation. (7 Marks)

b. Evaluate $\iint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS$ where $F=z\hat{i}+x\hat{j}-3y^2z\hat{k}$ and S is the surface of the cylinder $x^2+y^2=16$ in the first octant between $z=0$ and $z=5$. (7 Marks)

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