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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (S,FE) Examination January 2026 (2019 Scheme)

Course Code: MAT216

Course Name: MATHEMATICAL FOUNDATIONS FOR MACHINE LEARNING

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1	Let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \Re^2 : x.y = 0 \right\}$. Is V a vector space over $\Re$? Justify your answer?	3
2	Determine the following set of vectors in $\Re^3$ is linearly independent. $$v_1 = (1,0,1), v_2 = (2,-1,3),\ \ v_3 = (-1,2,-2)$$	3
3	Find the angle between the vectors; $\bar{a} = (2,-1,3)$, $\overline{b} = (1,4,-2)$	3
4	Find the eigenvalues of $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$	3
5	Let $f(x,y,z) = 2x^2 - 3y + z^3$. Find the gradient of the function at the point $(1,-2,1)$	3
6	Find the derivative of logistic sigmoid function using chain rule.	3
7	Let X be a continuous random variable with probability density function on $0 \le x \le 1$ defined by $f(x) = x^2$. Find the probability density function of $y = x^2$.	3
8	The time (in hours) required to repair a machine is exponentially distributed with parameter, $\lambda = \frac{1}{2}$. What is the probability that the repair time exceeds 2 hour	3
9	A firm manufactures pain relieving pills in two sizes A and B, size A contains 4 grains of element X , 7 grains of element Y and 2 grains of element Z , size B contains 2 grains of element X, 10 grains 0f element Y and 8 grains of Z. It is found by users that it requires at least 12grains of element X, 74 grains of element Y and 24 grains of element Z to provide immediate relief. It is required to determine that least number of pills a patient should take to get immediate relief. Formulate the problem as LPP	3
10	Check whether the function $f(x,y) = 3x^2 + 2xy + 4y^2 - 5x + 6y + 7$ is convex or not.	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Solve the following system of equations using Gauss elimination method: 7

$2x + 3y - z = 5; \quad 4x + 4y - 3z = 3\ ; \quad -2x + 3y + 2z = 7$

b) Find the inverse of the following matrix by 7

Gaussian elimination method $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

12 a) Consider the linear mapping ϕ: $\mathbb{R}^3 \rightarrow \mathbb{R}^4$ 7

$$\phi\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + 2x_2 + x_3 \\ x_1 + x_2 + x_3 \\ x_1 - 3x_2 \\ 2x_1 + 3x_2 + x_3 \end{bmatrix}$$

(a) Find the transformation matrix A_ϕ
(b) Determine the rank of A_ϕ
(c) Compute the kernel and image of ϕ
(d) What are the dimension of kernel and image of ϕ

b) Define vector space. Is $\mathbb{R}^2$ a vector space under operation $\bar{a} + \bar{b} = (x_1x_2,\ y_1y_2)$ 7
and $\alpha\bar{a} = (\alpha x_1, \alpha y_1)$ where $\bar{a} = (x_1, x_2)$, $\bar{b} = (y_1, y_2)$

Module -2

13 a) Find the singular value decomposition of matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ 7

b) Let V be the set of all continuous real valued functions defined on the closed interval $[0,1]$. Check whether V is an inner product space with inner product defined by $\langle f, g\rangle = \int_0^t f(t)g(t)dt$. 7

14 a) Use Cholesky decomposition to factorize the matrix $\begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix}$ 7

b) Find the orthonormal basis for $V_3(\mathbb{R})$ using Gram-Schmidt orthonormalization process with standard inner products for the basis $\{v_1, v_2, v_3\}$ where 7
$v_1 = (1,0,1),\ v_2 = (1,3,1)\ \textit{and}\ v_3 = (3,2,1).$

Module -3

15 a) Find the maximum and minimum values of the function 7

$f(x, y) = x^3 + y^3 - 3x - 12y + 20$

b) Let $f(x,y) = \begin{cases} \frac{yx^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ 7

1) Calculate the partial derivatives of f at $(0,0)$

2) Show that f is not differentiable at $(0,0)$

16 a) Let $f(x,y) = \sqrt{x^2 + y^2}$. 7

1) Find the linear approximation $L(x,y)$ at (3,4).

2) Using $L(x,y)$ estimate $f(3.01, 3.98)$.

b) Let $f(x,y) = \ln(1 + x + y)$. 7

1) Find the second order Taylor's series expansion of $f(x,y)$at the point (0,0)

2) Use the series to estimate the value of $f(0.05, -0.02)$ up to second order terms.

Module -4

17 a) The time (in minutes) between arrivals of customers at a bank follows an exponential distribution with $\lambda = 0.2$ 7

1) Write the probability density function of the distribution.

2) Find the probability that the next customer arrives within 3 minutes.

3) Find the expected time between arrivals and the variance.

b) Let the joint probability density function be 7

$f(x,y) = k(x + 2y);\ x = 1,2,3;\ y = 1,2,3.$

1) Find the value of k

2) Find the marginal distributions of X and Y

3) Find $P(X < 3, Y \geq 2)$

18 a) In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for 7

1) More than 2150 hours.

2) Less than 1950 hours.

3) More than 1920 hours and but less than 2160 hours.

b) There are two bags. The first bag contains four mangoes and two apples, the second bag contains four mangoes and four apples. We also have a biased coin, which shows heads with probability 0.6 and tails with probability 0.4. If the coin shows heads we pick a fruit at random from bag 2. Your flips the coin, picks a fruit at 7 random from the corresponding bag, and presents you a mango. What is the probability that the mango was picked from bag 2?

Module -5

19 a) Prove the negative entropy $f(x) = x\,log_2\,x$ is convex for $x > 0$ 7

b) Find the maximum and minimum values of $f(x, y) = xy$ subject to $x^2 + 2y^2 = 1$ using Lagrangian multiplier method. 7

20 a) Use the gradient descent method to find the minimum of the function 7

$f(x) = x^2 + xy + 10y^2 - 5x - 3y$ starting from $(-3, -1)$.

Perform two iterations.

b) Solve the Linear Programming Problem using simplex method; 7

Maximize $Z = 3x_1 + 5x_2$

subject to the contraints;

$2x_1 + 3x_2 \leq 12, \quad 2x_1 + x_2 \leq 8, \quad x_1, x_2 \geq 0$

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