

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (S,FE) (FT/WP) (S2 PT) Examination December 2025/January 2026 (2019 Scheme)

Course Code: MAT206**Course Name: GRAPH THEORY**

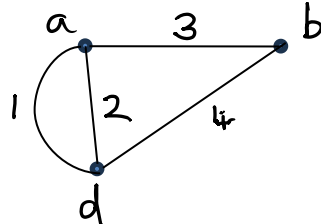
Max. Marks: 100

Duration: 3 Hours

PART A*(Answer all questions; each question carries 3 marks)*

Marks

- | | | |
|---|---|---|
| 1 | Prove that the sum of degree of all vertices in a graph is twice the number of edges | 3 |
| 2 | State Konigsberg bridge problem and draw its graph. | 3 |
| 3 | Define Euler graph and unicursal graph. Justify with an example that a connected graph is unicursal if it has exactly two vertices of odd degree. | 3 |
| 4 | Prove that in any digraph the sum of all indegrees is equal to the sum of all out degrees | 3 |
| 5 | Consider an electrical network containing 11 elements and 6 nodes. What is the minimum number of elements that must be removed so as to make the network circuitless? | 3 |
| 6 | Prove that the number of vertices in a binary tree is always odd. | 3 |
| 7 | Define fundamental circuit with an example. | 3 |
| 8 | What is a Planar graph? Give an example. | 3 |
| 9 | Write the circuit matrix of the following graph. | 3 |



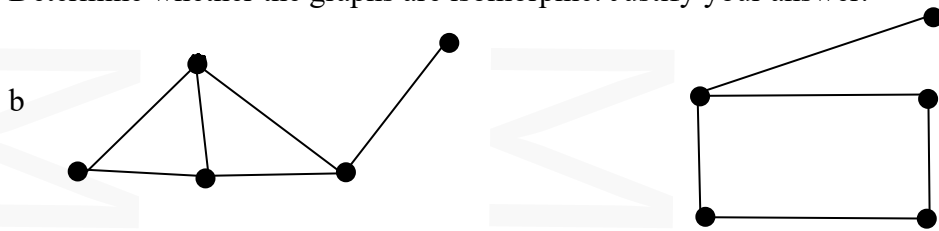
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|----|---------------------------|---|
| 10 | State Four color theorem. | 3 |
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PART B*(Answer one full question from each module, each question carries 14 marks)***Module -1**

- | | | |
|----|---|---|
| 11 | a) Define subgraph, edge disjoint subgraph and vertex disjoint subgraph with an example | 7 |
|----|---|---|

- b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 7

- 12 a) Determine whether the graphs are isomorphic. Justify your answer. 7



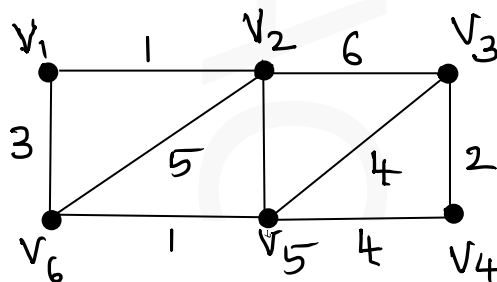
- b) What is the largest possible number of vertices in a graph G with 35 edges and all vertices are of degree at least 3? Give the answer in detail. 7

Module -2

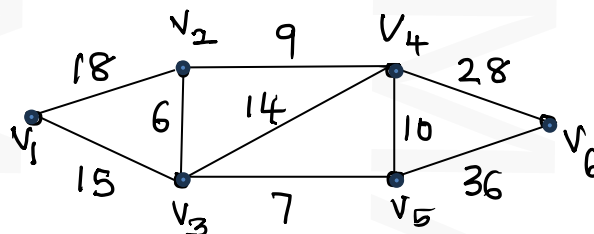
- 13 a) Prove that in a connected graph G with exactly $2k$ odd vertices, then there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph. 7
- b) Define directed walk, directed path and directed circuit in a digraph. Give example 7
- 14 a) Prove that a connected graph is an Euler graph if and only if all vertices of the graph are of even degree. 7
- b) Draw the digraph for the binary relation “is greater than” on $X=\{3,4,5,7,8\}$. Also write its relation matrix. 7

Module -3

- 15 a) Prove that a tree with n vertices has $(n-1)$ edges. 7
- b) Using Prim's algorithm find the minimum spanning tree for the graph below. 7



- 16 a) Draw all unlabelled trees on 6 vertices. 7
- b) Use Dijkstra's algorithm to find the shortest path between v_1 and v_5 . 7

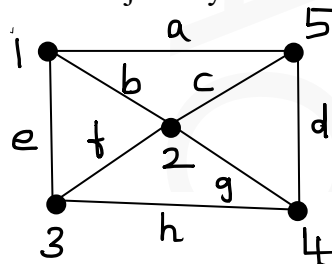


Module -4

- 17 a) Define edge connectivity. Prove that the edge connectivity of a graph cannot exceed the degree of the vertex with the smallest degree in G 7
- b) Prove that the Kuratowski's second graph $K_{3,3}$ is non-planar. 7
- 18 a) Prove that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G . 7
- b) Prove that a graph can be embedded on the surface of a sphere if and only if it can be embedded on a plane. 7

Module -5

- 19 a) Prove that every tree with two or more vertices is 2-chromatic 7
- b) Write the incidence matrix and adjacency matrix of the following graph. 7



- 20 a) If A and B are the incidence matrix and circuit matrix respectively of a graph G whose columns are arranged using the same order of edges prove that $AB^T = BA^T = 0 \pmod{2}$ 7
- b) Prove that covering g of a graph is minimal if and only if g contains no path of length three or more. 7
