QP Code:MA2T04C-3 **(Pages:4)**

Reg No : .... Name : ....

MAR ATHANASIUS COLLEGE OF ENGINEERING, KOTHAMANGALAM

(Govt. Aided Autonomous Institution)

Fourth Semester B.Tech. Degree Examination, May 2026

Course Code: B24MA2T04C

Course Name: GRAPH THEORY

Max. Marks: 100 Duration:3 Hours

PART - A: 30 Marks (10 Qns x 3 Marks)

Answer all questions

1. Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

2. Define walk, path and circuit in a graph with examples.

3. Let $X = \{10, 11, 12, 13, 14, 15\}$. Let R be the relation defined as "aRb iff |b-a| is a multiple of 3". Write the corresponding relation matrix.

4. Draw a graph with at least 5 vertices and is both Eulerian and Hamiltonian.

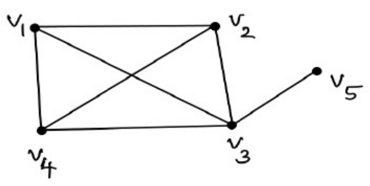
5. Show that the distance between pairs of vertices in a connected graph is a metric.

6. Define fundamental circuit with an example.

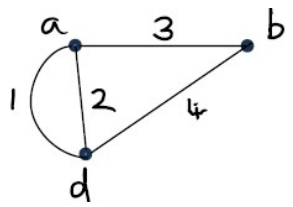
7. Define planar graphs. Is K_4 planar? Justify.

8. Draw a separable graph and non-separable graph.

9. Write the adjacency matrix for the following graph:



10. Write the circuit matrix of the following graph:



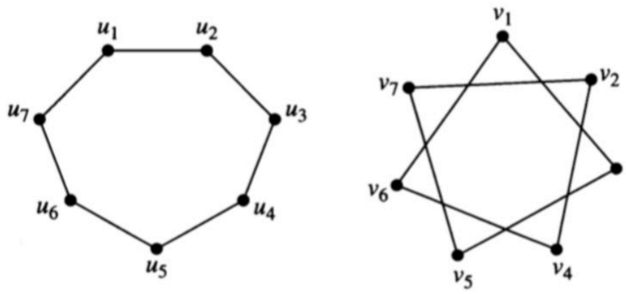
PART - B: 70 Marks (5 Qns x 14 Marks)

Answer any One full question from each module.

Module 1

11. a. Show that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (7 Marks)

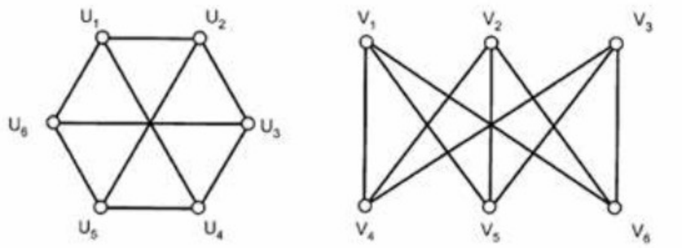
b. Are the following two graphs isomorphic? Justify your answer. (7 Marks)



OR

12. a. Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G with one end-vertex in V_1 and other end-vertex in V_2. (7 Marks)

b. Are the following graphs isomorphic or not? Justify your answer. (7 Marks)



Module 2

13. a. If G is a simple graph with n vertices such that for every vertex v, $d(v) \geq \frac{n}{2}$, then prove that G is Hamiltonian. (7 Marks)

b. Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. (7 Marks)

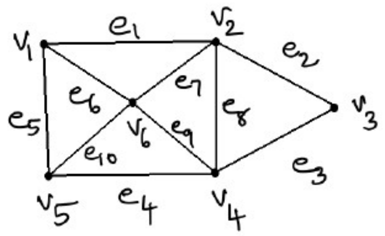
OR

14. a. Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can this arrangement last? Justify your answer (7 Marks)

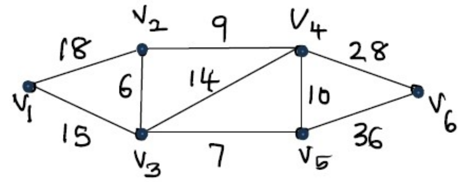
b. Define symmetric, asymmetric, complete symmetric and complete asymmetric digraphs with examples. (7 Marks)

Module 3

15. a. For any spanning tree of a connected graph with n vertices and e edges, prove that there are $n-1$ tree branches and $e-n+1$ chords. For the following graph, find two spanning trees and hence show that an edge that is a branch of one spanning tree can be a chord with respect to another spanning tree of the same graph. (7 Marks)



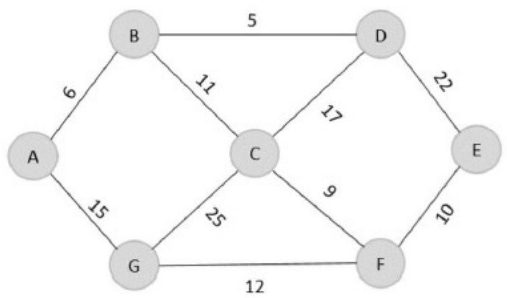
b. Use Dijkstra's algorithm to find the shortest path from v_1 to all other vertices (7 Marks)



OR

16. a. Prove that every tree has either 1 or 2 centres. (7 Marks)

b. Using Kruskal's algorithm, find the minimum cost spanning tree in the following graph: (7 Marks)



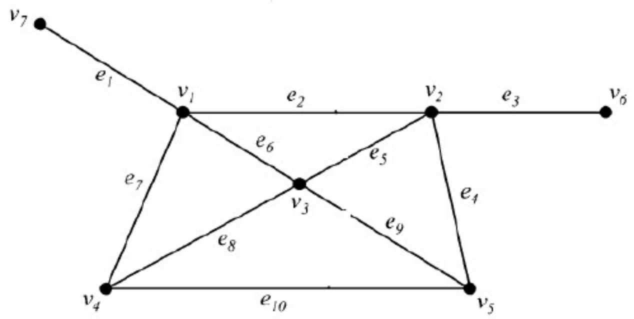
Module 4

17. a. Prove that every cut-set in a non-separable graph with at least two vertices will contain at least two edges. (7 Marks)

b. Prove that a vertex v in a connected graph G is a cut-vertex if and only if there exists 2 vertices x and y such that every path between x and y passes through v. (7 Marks)

OR

18. a. In the following graph, find a spanning tree and hence find all fundamental cut sets associated with it: (7 Marks)



b. For any graph G, prove the following inequality: (7 Marks)

vertex connectivity $\leq$ edge connectivity $\leq \frac{2e}{n}$

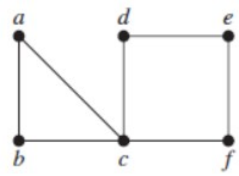
Module 5

19. a. Define semi-group, monoid and a group. Give different examples for each. Is the set of real numbers a group under multiplication? Justify. (7 Marks)

b. Let X(G) be the adjacency matrix of simple graph G. Then prove that (i,j)th entry of X^k gives the number of distinct edge sequences of length k between v_i and v_j. (7 Marks)

OR

20. a. Define path matrix and give two properties. Find the path matrix P(a,d) of the following graph after labelling the edges: (7 Marks)



b. Give an example for group of subgraphs. (7 Marks)

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