

Reg No : Name :

**MAR ATHANASIUS COLLEGE OF ENGINEERING, KOTHAMANGALAM**

(Govt. Aided Autonomous Institution)

Second Semester B.Tech. Degree Examination, May, 2026

Course Code: B24MA1T02

Course Name: ORDINARY DIFFERENTIAL EQUATIONS AND TRANSFORMS

(Common For All Branches)

Max. Marks: 100

Duration: 3 Hours

PART - A: 30 Marks (10 Qns x 3 Marks)**Answer all questions**

1. Solve $y''' + 9y' = 0$.
2. Find the Wronskian of $e^{-x} \cos 5x, e^{-x} \sin 5x$.
3. Examine the convergence of $\sum_{k=1}^{\infty} \frac{1}{2k-1}$.
4. Determine whether the series $\sum_{k=1}^{\infty} \frac{4^{k+2}}{7^{k-1}}$ converges and if so find its sum.
5. Find the Maclaurin series of $f(x) = \log(1+x)$ upto third degree terms.
6. Expand $f(x) = x^2$ in $0 < x < \pi$ as a half range sine series.
7. Express $f(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 < x < \pi \\ 0, & \text{if } x > \pi \end{cases}$ as a Fourier sine integral.
8. Derive the Fourier transform of $f(x) = \begin{cases} e^{-kx}, & x \geq 0, k > 0 \\ 0, & \text{otherwise} \end{cases}$.
9. Find the Laplace transform of $e^{-3t} \sin 3t$.
10. Find $L^{-1} \left[\frac{1}{(s-1)(s-2)} \right]$.

PART - B: 70 Marks (5 Qns x 14 Marks)**Answer any One full question from each module.****Module 1**

11. a. Solve the differential equation by method of undetermined coefficients $y'' + 4y = 8x^2$. (7 Marks)
b. Solve the IVP $y'' - 2y' + 5y = 0, y(0) = -3, y'(0) = 1$. (7 Marks)

OR

12. a. Solve the Euler-Cauchy equation $x^2 y'' + 7xy' + 13y = 0$. (7 Marks)
b. Using the method of variation of parameters solve $y'' + 4y = \tan 2x$. (7 Marks)

Module 2

13. a. Determine whether the alternating series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k}$ is conditionally convergent or not. (7 Marks)

- b. Test the convergence of the series a) $\sum_{k=1}^{\infty} \frac{2}{3^k + 5}$ (7 Marks)
- b) $1 - \frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \dots$.

OR

14. a. Show that the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k+3)}{k(k+1)}$ is conditionally convergent. (7 Marks)
- b. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{(k+3)!}{k!3^k}$. (7 Marks)

Module 3

15. a. Find the Fourier series of $f(x) = \begin{cases} \pi x; & 0 \leq x \leq 1, \\ \pi(2-x); & 1 \leq x \leq 2 \end{cases}$. (7 Marks)
- b. Find the half range Fourier cosine series of $f(x) = x$ in the interval $0 < x < \pi$. (7 Marks)
- Hence show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

OR

16. a. Find the Fourier series expansion of $f(x) = \begin{cases} \frac{\pi}{2} + x; & -\pi \leq x \leq 0, \\ \frac{\pi}{2} - x; & 0 \leq x \leq \pi \end{cases}$. (7 Marks)
- b. Find the half range sine series for $f(x) = e^x, 0 < x < 1$. (7 Marks)

Module 4

17. a. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$. (7 Marks)
- b. Find the Fourier sine transform of $f(x) = \begin{cases} k, & \text{if } 0 < x < a \\ 0, & x > a \end{cases}$. (7 Marks)

OR

18. a. Use Fourier integral to show that $\int_0^{\infty} \frac{\cos wx + w \sin wx}{1+w^2} dw = \begin{cases} 0, & x < 0 \\ \frac{\pi}{2}, & x = 0 \\ \pi e^{-x}, & x > 0 \end{cases}$. (7 Marks)
- b. Find the Fourier cosine transform of $f(x) = e^{-ax}, a > 0$. (7 Marks)

Module 5

19. a. Use Laplace transform to solve $y'' + 4y' + 13y = 2e^{-t}, y(0) = 0, y'(0) = -1$. (7 Marks)
- b. Evaluate $L^{-1} \left[\frac{2s+1}{s^2+2s+5} \right]$. (7 Marks)

OR

20. a. Using convolution theorem, find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$. (7 Marks)
- b. Find the Laplace transform of a) $\int_0^t e^{-4t} \sin 3t \cos 2t dt$ b) $(\sin t + \cos t)^2$. (7 Marks)
